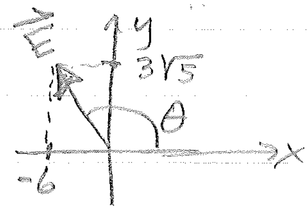
Physics 302 Photonics
Homework 2 - Chapter 3
SOLUTIONS

(3.5) $\vec{E} = (-6\hat{i} + 3\sqrt{5}\hat{j})(10^4 \frac{V}{m}) \exp\left\{i\left[\frac{1}{3}(\sqrt{5}x + 2y)\pi \times 10^7 - 9.42\times10^{15} t\right]\right\}$

(a) direction of $\vec{E}$



$\tan\theta = \frac{3\sqrt{5}}{-6} = -\frac{\sqrt{5}}{2}$

$\boxed{\theta = 132^\circ \text{ wrt } x\text{-axis}}$

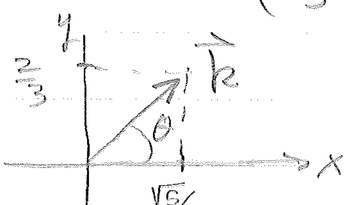
(b) Amplitude $= E = \left|(-6\hat{i} + 3\sqrt{5}\hat{j}) * 10^4 \frac{V}{m}\right|$

$E = \sqrt{(-6)^2 + (3\sqrt{5})^2} * 10^4 \, V/m = \boxed{9\times10^4 \frac{V}{m} = E}$

(c) direction of propagation $\vec{k}$ is given by phase

$\phi = \vec{k}\cdot\vec{r} - \omega t$

$\vec{k} = \vec{\nabla}\phi = \left(\frac{\sqrt{5}}{3}\hat{i} + \frac{2}{3}\hat{j}\right)\pi * 10^7$



$\tan\theta = \frac{2/3}{\sqrt{5}/3} = \frac{2}{\sqrt{5}}$

$\boxed{\theta = 42^\circ \text{ wrt } x\text{-axis}}$

Note: This is orthogonal to $\vec{E}$

(d) $k = \sqrt{\left(\frac{\sqrt{5}}{3}\right)^2 + \left(\frac{2}{3}\right)^2}\, \pi\cdot10^7 = \sqrt{\frac{5+4}{9}}\, \pi\cdot10^7$

$\boxed{k = \pi * 10^7 / m}$ also $k = \frac{2\pi}{\lambda} \Rightarrow \boxed{\lambda = 2\times10^{-7} m}$

(e) $\boxed{\omega = 9.42\times10^{15} \frac{rad}{s}}$ also $\omega = 2\pi\nu \Rightarrow \boxed{\nu = 1.50\times10^{15} Hz}$

(f) $\boxed{v = \omega/k = 3.00\times10^8 \, m/s}$

(3.14) $I = d\mathcal{P}/dA$

where $\mathcal{P}$ = power.

For I = constant $\Rightarrow \mathcal{P} = \int I\,dA = I \cdot 4\pi r^2$

so $I = \dfrac{\mathcal{P}}{4\pi r^2} = \dfrac{20\,W}{4\pi(1.00\,m)^2} = \boxed{1.59\,\dfrac{W}{m^2} = I}$

(3.19) $I = \dfrac{c\epsilon_0}{2} E_0^2$

$$E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2\times10^{20}\,W/m^2}{(3.00\times10^8\,\frac{m}{s})(8.85\times10^{-12}\,\frac{C^2 s^2}{m^3 kg})}}$$

$$\boxed{E_0 = 2.74\times10^{11}\,\frac{V}{m}}$$

(3.23) $\dfrac{\#\text{ photons}}{\text{sec}} = \dfrac{\mathcal{P}}{h\nu} = \dfrac{\mathcal{P}}{hc/\lambda}$

$$= \frac{(100\,\frac{J}{s})(550\,nm)}{(6.63\times10^{-34}\,J\cdot s)(3.00\times10^8\,m/s)}$$

$$\boxed{\frac{\#\text{ photons}}{s} = 2.76\times10^{20}\ \text{photons}/s}$$

(3.25) $I = \mathcal{P}/4\pi r^2 = \dfrac{100\,W}{4\pi(1m)^2} = \boxed{7.96\,\dfrac{W}{m^2} = I}$

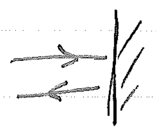
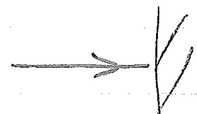
$$E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \boxed{77.4\,\frac{V}{m} = E_0}$$

$$B_0 = \frac{E_0}{c} = \boxed{2.58\times10^{-7}\,\frac{Wb}{m^2} = B_0}$$

(3.27) $p = \hbar k = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} = h\nu/c$

$$p = \frac{(6.63 \times 10^{-34}\ J \cdot s)(1. \times 10^{19}/s)}{3.00 \times 10^{8}\ m/s}$$

$$\boxed{p = 2.21 \times 10^{-23}\ \frac{kg \cdot m}{s}}$$

(3.32)  

$(1-\alpha)$ are 100% reflected — α are 100% absorbed

(change in momentum) $= \Delta p = 2\hbar k$; $\Delta p = \hbar k$

The pressures from these two types of incidence are additive:

$$\text{pressure} = P = \frac{F}{A} = \frac{1}{A}\frac{\Delta p}{\Delta t}$$

$$\frac{\#\text{photons hitting}}{\text{time} \cdot \text{area}} = I/h\nu$$

$$P = (1-\alpha)(2\hbar k)\frac{I}{h\nu} + \alpha(\hbar k)\frac{I}{h\nu}$$

$$P = \frac{\hbar k I}{h\nu}(2-\alpha) = \frac{(h\nu/c) I}{h\nu}(2-\alpha)$$

$$\boxed{P = \frac{I}{c}(2-\alpha)}$$